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retain only the leading terms of the expansion. Retaining for the present the first two terms, the preceding expression becomes

$$\frac{2d\lambda}{\lambda^2}\cos\left\{\frac{2\pi}{\lambda}(x-ct-a)\right\}\int_0^{b-a}f(a+y)dy$$

$$+\frac{2d\lambda}{\lambda^2}\,\frac{2\pi}{\lambda}\sin\left\{\frac{2\pi}{\lambda}(x-ct-a)\right\}\int_0^{b-a}yf(a+y)dy$$

The first of these integrals vanishes in consequence of the condition that the pulse consists of compensating positive and negative parts; the second integral depends on the particular form of the pulse, but is independent of λ, and will in general have a finite value different from zero, which we shall denote by C. The spectroscopic element of the pulse with wave-lengths between λ and $\lambda + d\lambda$ is therefore

$$\frac{4\pi C d\lambda}{\lambda^3}\sin\left\{\frac{2\pi}{\lambda}(x-ct-a)\right\}$$

Now if in any disturbance the spectroscopic element with wave-lengths between λ and $\lambda + d\lambda$ is

$$f(\lambda)\,.\,d\lambda\,.\,\sin\left\{\frac{2\pi}{\lambda}(x-ct-\epsilon)\right\},$$

it is known * that the energy radiated in this interval is proportional to $\lambda^2\{f(\lambda)\}^2 d\lambda$. So in the present case the energy radiated with wave-lengths between λ and $\lambda + d\lambda$ is proportional to $\lambda^{-4}d\lambda$. As the various pulses are supposed to be entirely unco-ordinated as regards phase, this law which holds for each of them individually will hold also for their aggregate; and therefore, in the radiation emitted by any body, the energy in the part of the spectrum between λ and $\lambda + d\lambda$ is proportional to $\lambda^{-4}d\lambda$ in the region of longer wave-lengths.

From this result, by an application of the thermodynamical theorem (II.) above, we immediately deduce the consequence that the radiation of a body at temperature T is, in the ultra-red, proportional to $T\lambda^{-4}d\lambda$.

Law (III.) is thus established as a direct consequence of the nature of white light, without reference to the mechanism of radiation in the radiating body.

On the Resolving Power of Spectroscopes. By E. T. Whittaker, Sc.D., F.R.S., Royal Astronomer of Ireland.

The resolving power of a spectroscope is, according to the usual definition, the value of $\frac{\lambda}{\delta\lambda}$, where $\delta\lambda$ is such that the luminous centre of the spectral line of wave-length $\lambda + \delta\lambda$ falls on the first minimum of intensity of the line of wave-length λ. It is a well-

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* *Cf.* Lord Rayleigh, *Phil. Mag.*, xxvii. 460 (1889).

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known theorem that if θ denotes the deviation of light of wave-length λ, so that $\frac{d\theta}{d\lambda}$ measures the dispersion caused by the spectroscope, and if a denotes the breadth of the beam of parallel light of the wave-length λ as it emerges from the dispersive apparatus, then the resolving power of the instrument is $a\frac{d\theta}{d\lambda}$.

The resolving power of a spectroscope can also be discussed in connection with the modern view of the nature of luminous disturbance as a succession of pulses. This has been done by several writers.* In the case of the grating, it is evident that a single incident pulse is broken up into as many separate pulses as there are spacings in the grating: these separate pulses, as they issue from the grating, are no longer in the same wave-front, and they will consequently follow each other at regular intervals to the focus of the observing telescope. In this way we can establish for the grating a theorem to the effect that the number of alternations, in the disturbance which is formed from a single original pulse, is equal to the resolving power of the instrument.

The case of dispersion by a prism presented greater difficulties, as it seemed somewhat mysterious that alternations should be formed at all in this case. But the discussions of Professors Schuster and Ames and Lord Rayleigh † make it evident that the difference between the wave-velocity and the group-velocity of luminous disturbance in the material of the prism will cause a single incident pulse to be spread out into an alternating disturbance; and the working out of this idea verifies the same theorem for prisms as has been stated above for gratings, namely, that the number of pulses into which a single incident pulse is spread out is equal to the resolving power of the instrument.

It becomes, therefore, of interest to determine whether this theorem can be established in the most general case by a proof which will be independent of the nature of the dispersive apparatus, and consequently applicable to every type of spectroscope. The object of the present note is to communicate such a proof.

Suppose, then, that a pulse, which for our present purpose we may regard as a thin plane sheet of disturbance moving perpendicularly to its own plane, is incident on any dispersive apparatus. For simplicity we shall suppose that the Fourier analysis of the pulse will yield radiations of wave-lengths λ and $\lambda + d\lambda$ only. At incidence the *pulse-front*, or plane on which the disturbance exists, is the same as the *phase-front*, or plane parallel to which the λ-disturbance is in the same phase; but after emergence from the dispersive apparatus this will no longer in general be the case. Suppose, then, that ϕ denotes the angle between the pulse-front and phase-front on emergence; and let $d\theta$ denote the angle between the phase-fronts for the λ radiation and the $\lambda + d\lambda$ radiation, so that $d\theta$ is the dispersion. If, now, A C, B E in the figure are two successive crests of the λ-disturbance at emergence, and A D, B F

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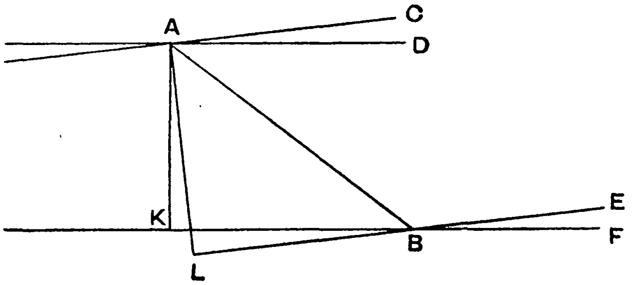
* *Cf. e.g.* Lord Rayleigh, Larmor, Schuster, and Ames.

† Schuster, *Theory of Optics*, p. 325 (1904); Ames, *Astrophysical Journal*, July 1905; Rayleigh, *Phil. Mag.*, x. p. 401 (1905).



two successive crests of the $\lambda + d\lambda$ disturbance, then the pulse-front will pass through the points A,B, where the crests reinforce each other. Thus if A K, A L are perpendicular respectively to B F and B E, we have

$$AK = \lambda, \qquad AL = \lambda + d\lambda, \qquad K\hat{A}L = d\theta, \qquad ABK = \phi$$

and so from the figure

$$AB \sin \phi = \lambda, \qquad AB \sin (\phi + d\theta) = \lambda + d\lambda$$

Subtracting these two equations, we have

$$AB \cos \phi \; d\theta = d\lambda$$

and dividing this into the first of them, we have

$$\tan \phi = \lambda \frac{d\theta}{d\lambda}$$

This equation, which is applicable to any dispersive apparatus, shows that the tangent of the angle between the pulse-front and the phase-front at emergence is equal to the product of the wave-length into the dispersion.

Since the resolving power is the product of the dispersion into the breadth of the emergent beam, it follows from this equation that the resolving power is equal to the product of $\frac{1}{\lambda} \tan \phi$ into the breadth of the emergent beam, *i.e.* it is equal to the product of $\frac{1}{\lambda} \sin \phi$ into the breadth of the emergent pulse. But $\frac{\lambda}{\sin \phi}$ is evidently A B in the figure, *i.e.* it is the interval between two successive maxima of intensity in the pulse-front; and thus the resolving power is equal to the quotient of the breadth of the emergent pulse by the interval between two successive maxima in it; that is, the resolving power is equal to the number of separate maxima of disturbance in the emergent pulse-front; which was to be proved.

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Extended Nebula near 26 *Ceti.* By Dr Max Wolf, Assoc. R.A.S.

On four plates taken with the Bruce 16-inch telescope in September I find an extended nebula, which attracts attention both by its situation and its appearance.

Practically all extended nebulosities are found in or near the Milky-way, but this object is about 70° distant from the plane of the Milky-way. Its densest part is situated at

$$\alpha = 0^h\ 57^m{\cdot}4 \qquad \delta = +1^\circ\ 20' \qquad (1885{\cdot}0)$$

Round this is spread faint nebulous matter of varying intensity. At several points small clouds of greater density shine forth. Further out, the intensity is so feeble that it is impossible to exactly define its limits. It seems that these will be materially altered by lengthening the exposure. My exposures extend to four hours. With these the nebulous cloud extends about 40′ in declination and about 30′ in R.A. The three B.D. stars

	$\alpha =$ h m	$\delta =$
$+1^\circ.191$	$\alpha = 0\ 56{\cdot}4$	$\delta = +1^\circ 24'$
1 .200	57·0	1 27
1 .201	57·3	1 32

are all involved in the cloud.

Southwards it reaches nearly to the B.D. star

$$+0^\circ.207 \qquad 0^h\ 56^m{\cdot}9 \qquad +1^\circ 0'$$

but it cannot be certainly detected up to the star. Especially towards the west the nebulous matter promises to go much further.

Under the microscope the brighter parts of the mass are filled with numerous very small spots and short trails, so that the appearance is very different from that presented by the Milky-way nebulæ. It seems possible that this object is a multitude of very small planetary nebulæ collected in a cluster, which a more powerful instrument than mine may perhaps resolve.

Astrophysical Observatory, Heidelberg:
1906 *October.*

Observations of the Satellite of Neptune from Photographs taken at the Royal Observatory, Greenwich, between 1905 *December* 19 *and* 1906 *April* 25.

(*Communicated by the Astronomer Royal.*)

The following measures of position-angle and distance of Neptune's satellite were made from photographs taken with the 26-inch refractor of the Thompson equatorial. The occulting shutter was used as in previous years. The photographs were